RF Linac for High-Gain FEL

RF Linac

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US Particle Accelerator School
June 16-20, 2014

LA-UR 14-23985

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RF Acceleration

Lorentz force caused by an electric field acting along the direction of motion changes the beam's energy

$$\Delta W = \int \mathbf{F} \cdot d\mathbf{s} = -\int e\mathbf{E} \cdot d\mathbf{s}$$

To accelerate charged particles, the RF wave must have electric fields along the direction of propagation of the particle and the wave itself. However, EM waves in free space only have electric field that is transverse to direction of propagation.

To get non-zero acceleration of charged particles co-propagating with the electromagnetic wave, we have to do the following:

1. Use a resonant cavity that has transverse magnetic (TM) modes. The TM_{010} mode has axial electric field to accelerate particles along the axial direction.

2. Load the cavity with disk-and-washers to slow the phase velocity of the RF wave to the speed of light *c*, so that a charged particle traveling at speed slightly less than *c* will have non-zero acceleration accumulated over many RF cycles.

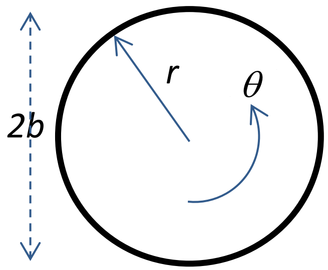
EM Waves in Cylindrical Waveguides

Wave equation for cylindrical waveguides (z = wave propagation direction)

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 0$$

Select solution of the form $E_z(r,t) = R(r)\exp[i(kz - \omega t)]$

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} + \left(\frac{\omega^2}{c^2} - k^2\right)R = 0$$



For the wave to propagate, the term in parentheses has to be > 0.
More generally, solutions exist for both the transverse electric (TE_{nm}) and transverse magnetic (TM_{nm}) modes, where

n = number of full-period variations over 2π in θ

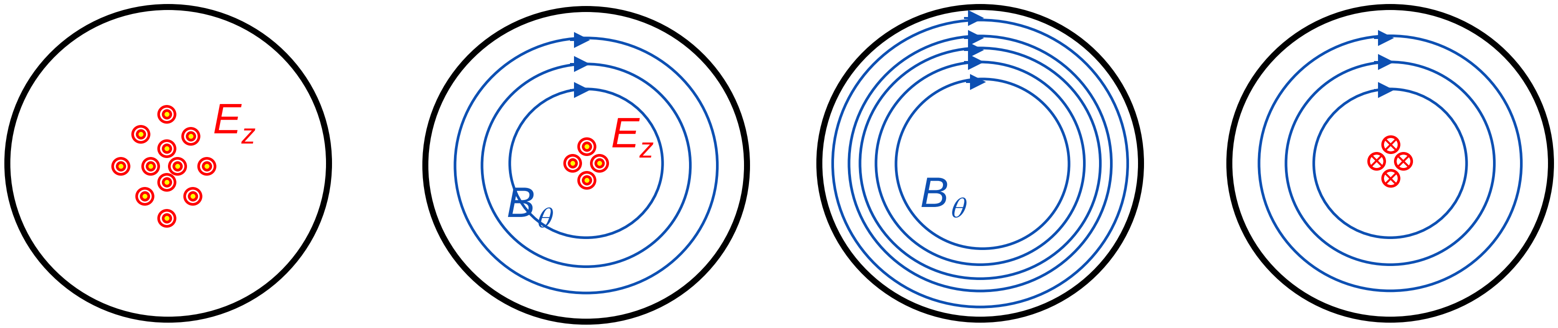
m = number of nodes along the radial direction r

These modes have different cut-off frequency. The TM_{01} mode, which has one radial node at $r = b$, is used for RF acceleration. Its cut-off frequency is

$$\omega_{01} = 2.405\frac{c}{b}$$

TM_{01} Mode in Circular Waveguides

Snapshots of E_z and B_θ fields on the WG cross-section at four different times



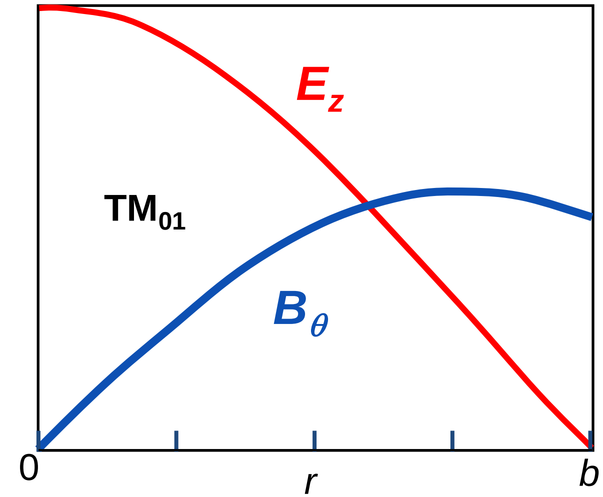
$t = 0$ $\quad t = \frac{\pi}{4}$ $\quad t = \frac{\pi}{2}$ $\quad t = \frac{3\pi}{4}$

Longitudinal electric field peaks on axis

$$E_z(r,t) = E_0 J_0(k_c r) e^{i(kz-\omega t)}$$

Electric field is 0 at $r = b$, where $k_c r = 2.405$

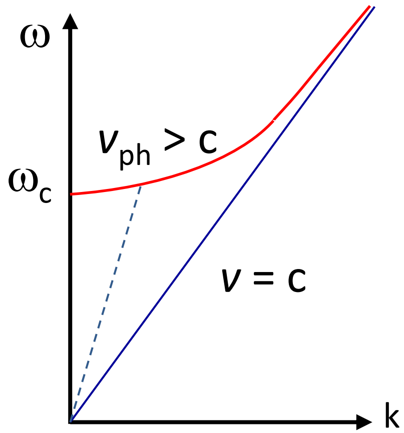
$$B_\theta(r,t) = \frac{iE_0}{c} J_1(k_c r) e^{i(kz-\omega t)}$$



Magnetic field peaks at $r = 0.77b$, where $k_c r = 1.804$

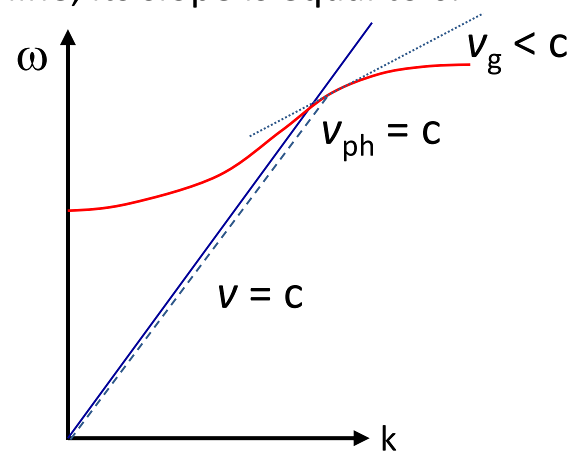
Dispersion Diagram

In the plot of ω vs k, waves with frequencies below cut-off frequency ω_c do not propagate. Those above ω_c follow the dispersion relation and propagate with v_{ph} greater than *c* in unloaded structures (left). In loaded structures (right), the dispersion curve rolls over and a frequency exists where v_{ph} = c. The slope of the dispersion curve is the group velocity (which is always less than *c*). The solid blue line indicates the "light" line; its slope is equal to *c*.



dispersion relation

$$\omega^2 = \omega_c^2 + (kc)^2$$



In unloaded RF structures, v_{ph} is always greater than c, and the co-propagating particles experience zero time-averaged energy gain.

Loading the structure with disk-and-washers slows down the wave and at the frequency where v_{ph} = c, relativistic particles can be accelerated by riding the wave.

Phase velocity

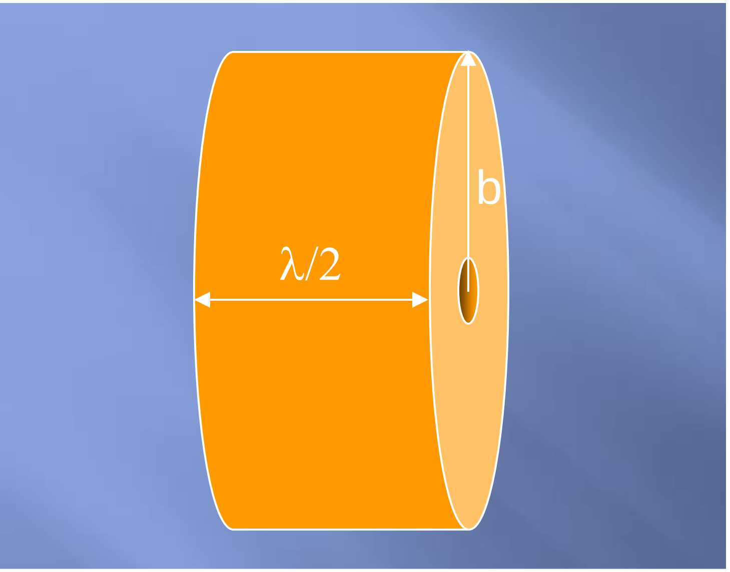
$$\upsilon_{ph} = \frac{c}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

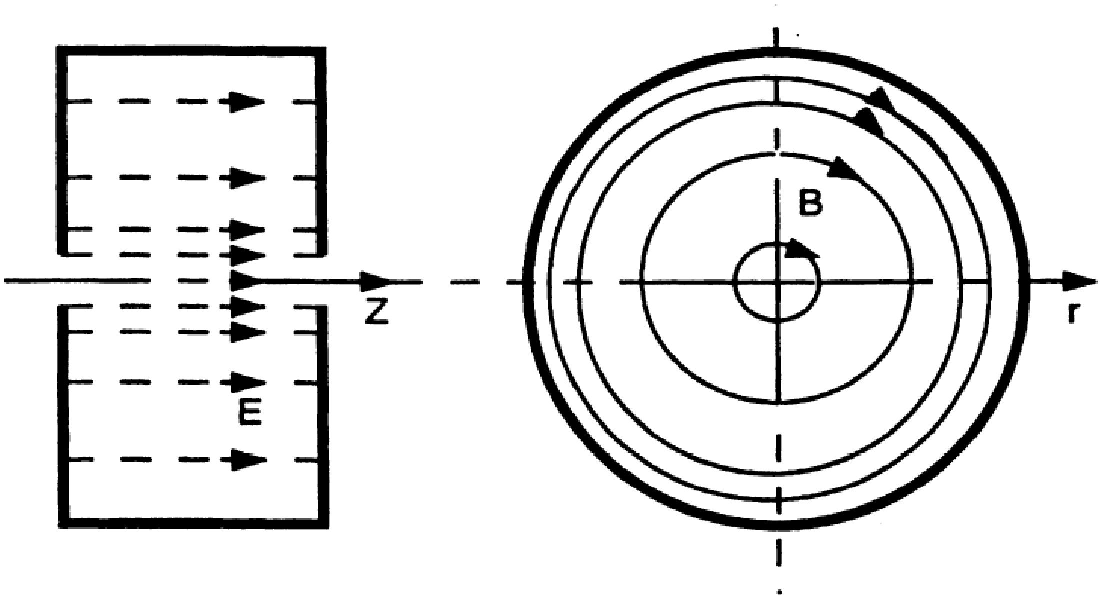
Group velocity

$$\upsilon_g = \frac{d\omega}{dk}$$

$$\upsilon_{ph} = \frac{\omega}{k_0 + n\left(\frac{2\pi}{d}\right)}$$

TM_{010} Mode in Pillbox Cavity





$$E_z(r,t) = E_0 J_0(k_c r)\cos(\omega t)$$

$$E_r(r,t) = -\frac{E_0}{c}\sqrt{1-\frac{\omega_c^2}{\omega^2}}\,J_1(k_r r)\sin(\omega t)$$

$$B_\theta(r,t) = -\frac{E_0}{c}J_1(k_c r)\sin(\omega t)$$

$$E_\theta = B_r = B_z = 0$$

A pillbox cavity consists of a short cylindrical waveguide capped at both ends with conducting plates. The lowest-frequency TM mode is TM_{010} with cell length = $\lambda/2$.

TM_{010} resonance frequency

$$\omega_{01} = 2.405\frac{c}{b}$$

Energy Gain and Chirp

Accelerating Field

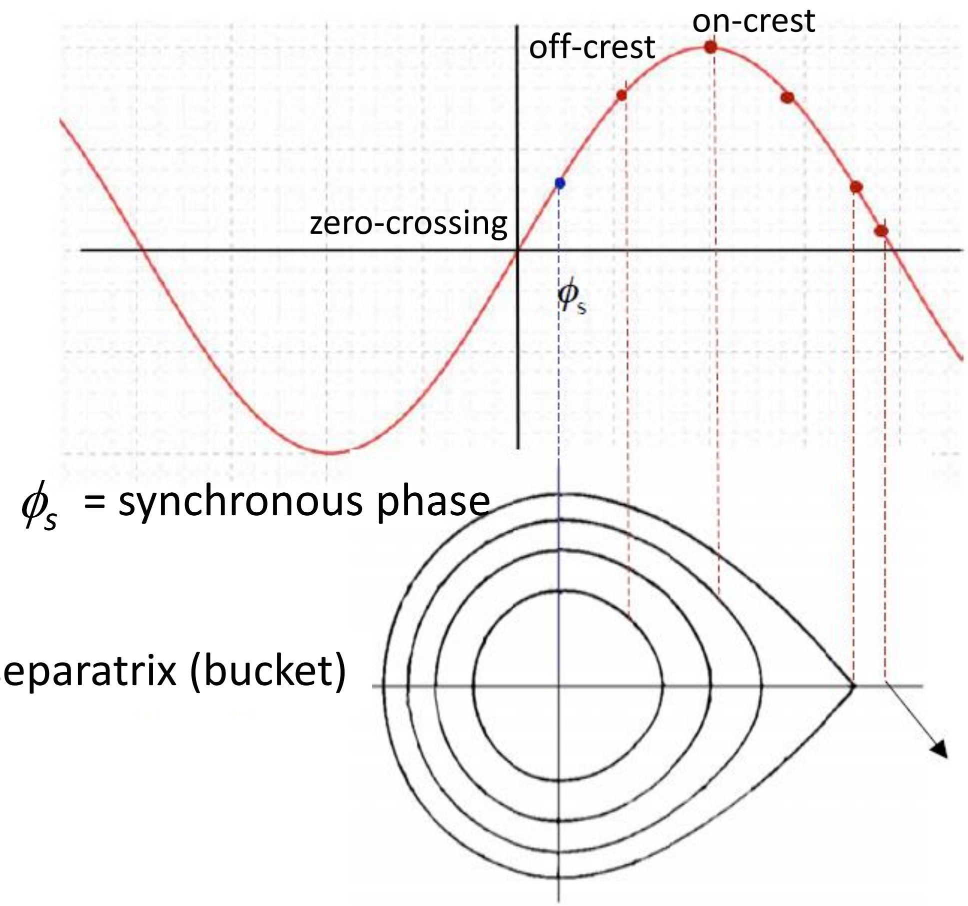
$$E_z(t) = E_0 \cos(\omega t + \phi)$$

Energy Gain

$$\boxed{\Delta W = eE_0 TL \cos\phi_s}$$

Energy Chirp

$$\frac{d(\Delta W)}{d\phi} = -eE_0 TL \sin\phi_s$$



RF separatrix (bucket)

Maximum energy gain occurs on-crest (ϕ_s = 0)

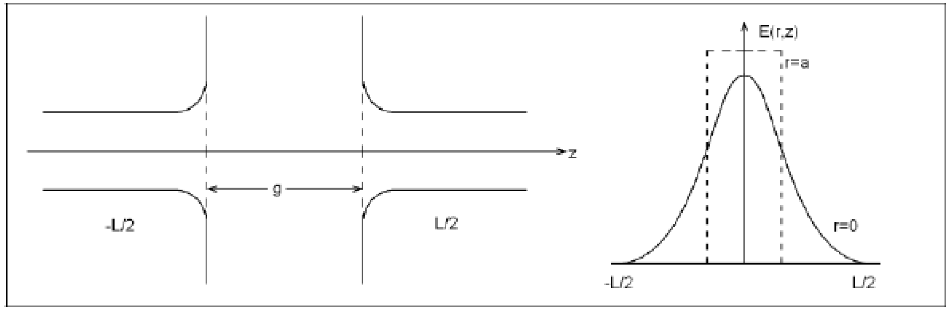
Off-crest acceleration with ϕ_s between $-\pi/2$ and 0 gives rise to energy chirp

Transit Time Factor

Transit Time Factor: ratio of energy gain in a time-varying field to a DC field

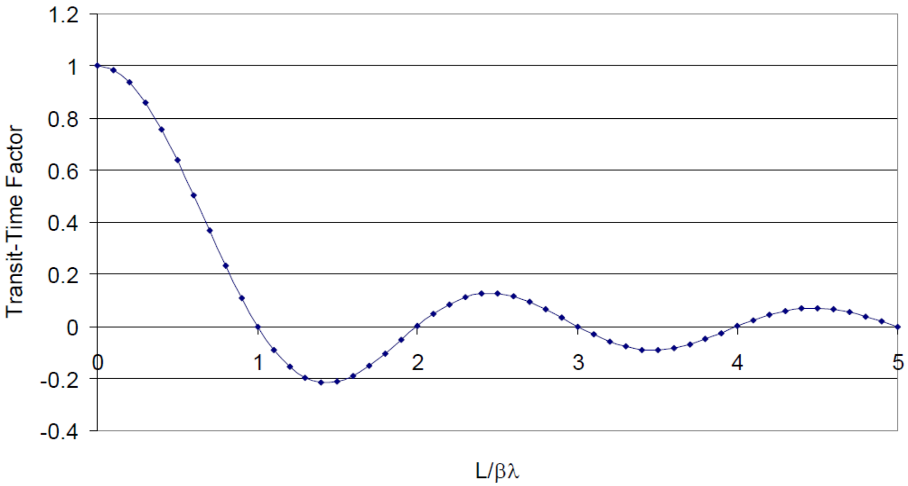
Transit Time Factor

$$T = \frac{1}{V_0} \int_{-L/2}^{L/2} E_0(r,z)\cos(kz)dz$$



Assuming a square-wave field across the gap length and radius

$$T = \frac{\sin\left(\frac{\pi L}{\lambda}\right)}{\left(\frac{\pi L}{\lambda}\right)}$$



The real TTF has a Bessel I_0 term to account for its radial dependence

Accelerating Gradients

Accelerating gradient: maximum energy gain divided by the length of an RF cell.

Average gradient: the product of accelerating gradient and transit time factor.

Real-estate gradient: final beam energy divided by the total length of the linac and other components such as the vacuum vessels, power couplers, etc.

Why do we care about accelerating gradients?

High gradients translate into

- short linac length and thus lower costs
- bright electron beams
- stronger RF focusing
- higher RF power is needed
- more cooling (either water or liquid helium) is needed

Power Dissipation

Power dissipation in the cavity walls due to ohmic ($I^2 R$) heating

$$P = \frac{R_S}{2} \int \left| H \right|^2 dA$$

For NCRF, power dissipation means the RF sources (klystrons) have to provide the necessary peak power AND we have to remove heat from the RF cavities.

<u>Shunt impedance</u>: high shunt impedance $\rightarrow$ low RF power consumption.

Optimize NCRF cavity shapes for high shunt impedance and TTF

For SCRF, power dissipation translates into heat load at cryogenic temperatures. Cryogenic heat load increases the size of the helium cryoplant.

<u>Cavity Q_0</u>: The higher the unloaded Q, the lower the cryogenic load.

Prepare cavity surfaces and optimize cavity shapes to maximize Q_0.

RF Considerations

NCRF

RF power dissipation per unit length

$$\frac{P}{L} = \frac{(E_0 T)^2}{r_{sh}}$$

Shunt impedance per unit length

$$r_{sh} \approx 1.28 \frac{\mathrm{M}\Omega}{m} \sqrt{f[MHz]}$$

Cavity unloaded Q

$$Q_0 \approx \frac{270\Omega}{R_S} \approx 3\times10^4$$

Surface Resistance

$$R_S = \frac{1}{\sigma\delta} \approx 10m\Omega$$

SCRF

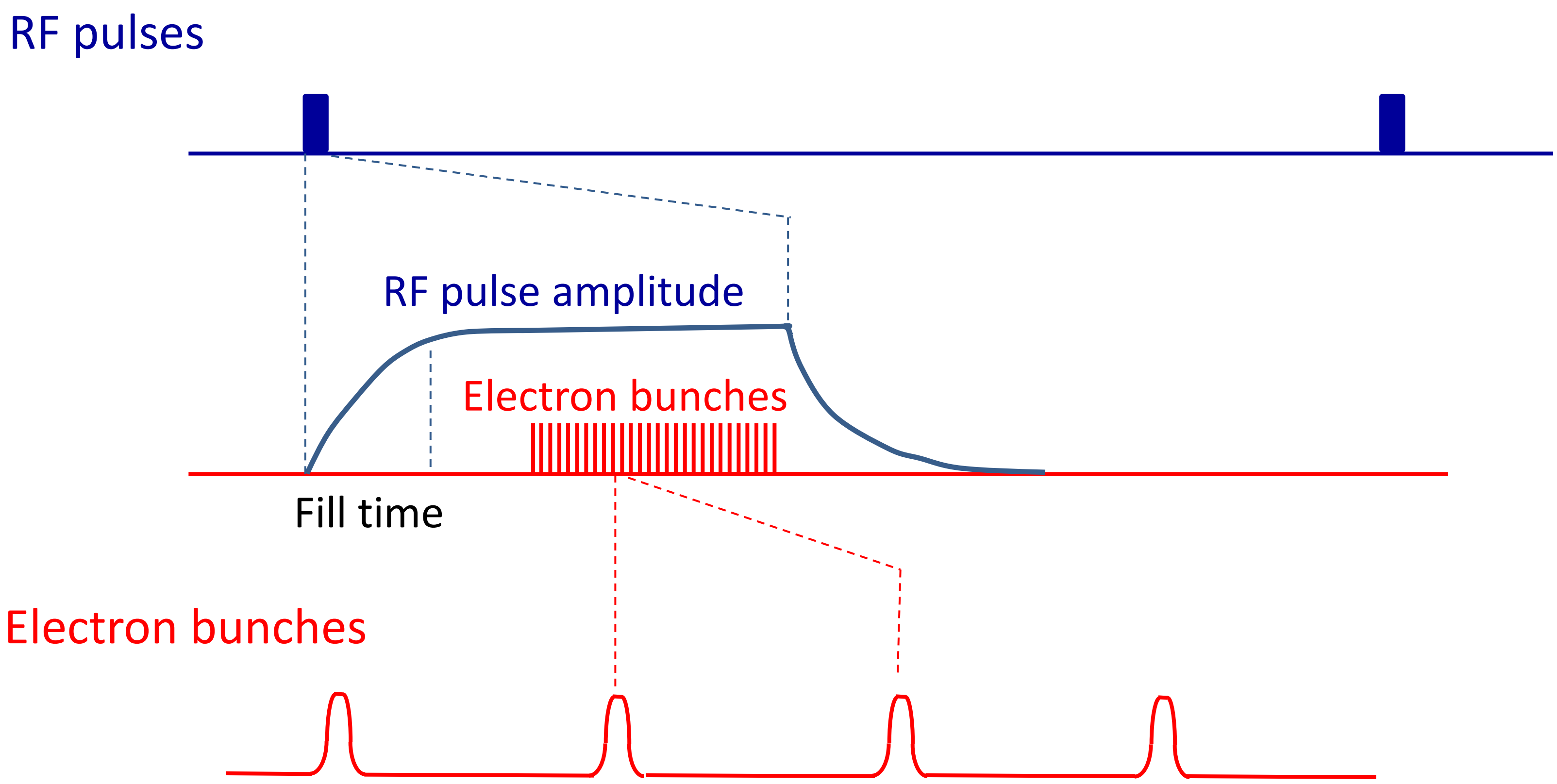
$$\frac{P}{L} = \frac{(E_0 T)^2}{\left(\frac{r}{Q}\right) Q_0}$$

$$\frac{R}{Q} \approx \frac{100\Omega}{cell} \qquad \frac{r}{Q} = \frac{R}{Q} \frac{\# cells}{m}$$

$$Q_0 \approx \frac{270\Omega}{R_S} \approx 3\times10^{10}$$

$$R_S = R_{BCS} + R_{residual} \approx 10n\Omega$$

RF-linac Pulse Format



Bunch separation = n / f

n is the number of RF buckets between electron bunches

Pulse Format Considerations

- **Normal-conducting RF Linac**

  Traveling-wave linac

Example: S-band SLAC, C-band SACLA	Water-cooled copper Accelerating gradient ~ 30 MV/m Maximum RF pulse ~ 3 μs Fill time < 1 μs

  Standing-wave linac

Example: L-band ISIR (Osaka)	Water-cooled copper Accelerating gradient ~ 20 MV/m Maximum RF pulse ~ 30 μs Fill time ~ 2 μs

- **Super-conducting RF Linac**

Example: L-band TESLA (DESY)	Liquid He cooled niobium Accelerating gradient ~ 15 MV/m Long pulse to continuous-wave (CW) Fill time ~ 500 μs

RF Bands & Wavelengths

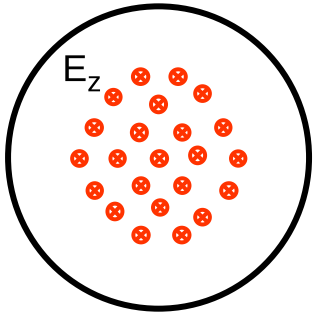
	Frequency range	Common frequency	Wavelength
VHF	30 – 300 MHz	187 MHz	160 cm
UHF	0.3 – 3 GHz	500 MHz	59.96 cm
L-band	1 – 2 GHz	1.3 GHz	23.06 cm
S-band	2 – 4 GHz	2.856 GHz	10.50 cm
C-band	4 – 8 GHz	5.712 GHz	5.248 cm
X-band	8 – 12 GHz	11.424 GHz	2.624 cm
Ku-band	12 – 18 GHz	17.14 GHz	1.75 cm

Stored Energy

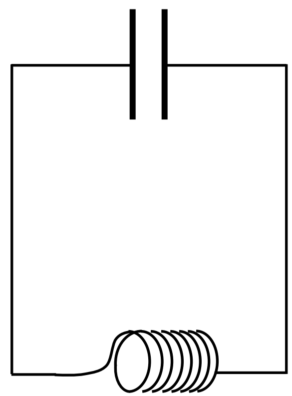
The stored energy alternates between electric and magnetic fields

Electric field

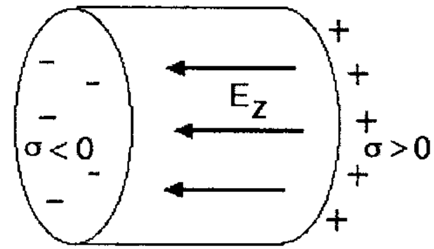
$$U = \frac{\varepsilon_0}{2}\left|E_0\right|^2$$



capacitor



inductor

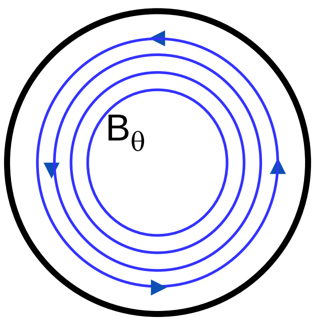


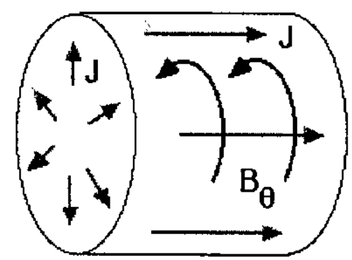
Charge

$$U = \frac{1}{2} QC^2$$

Magnetic field

$$U = \frac{\mu_0}{2}\left|B_0\right|^2$$





Current

$$U = \frac{1}{2} IL^2$$

and scales with the square of the accelerating gradient.

$$U = \frac{\varepsilon_0}{2}\left|E_0\right|^2 J_1^2(2.405)\mathrm{V}_{cavity}$$

Equivalent Lump Circuit

- **Resonance Frequency**

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- **Shunt Impedance**

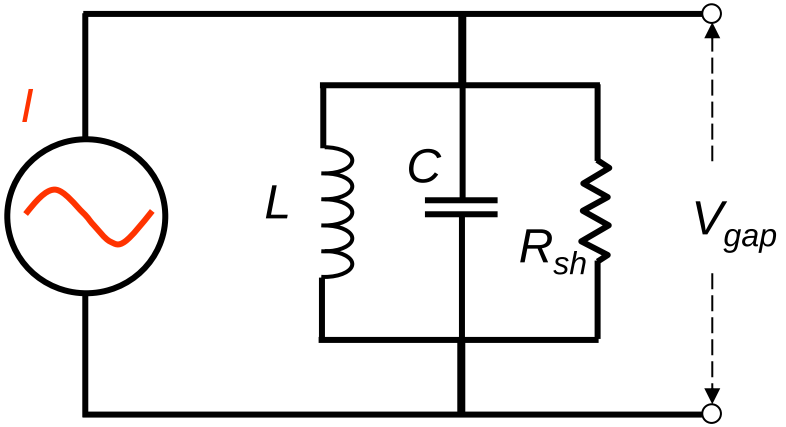
$$R_{sh} = \frac{\left|V_{gap}\right|^2}{P}$$

- **Cavity Q**

$$Q_0 = \frac{\omega_0 U}{P}$$

- **R/Q**

$$\frac{R}{Q} = \frac{\left|V_{gap}\right|^2}{\omega_0 U}$$



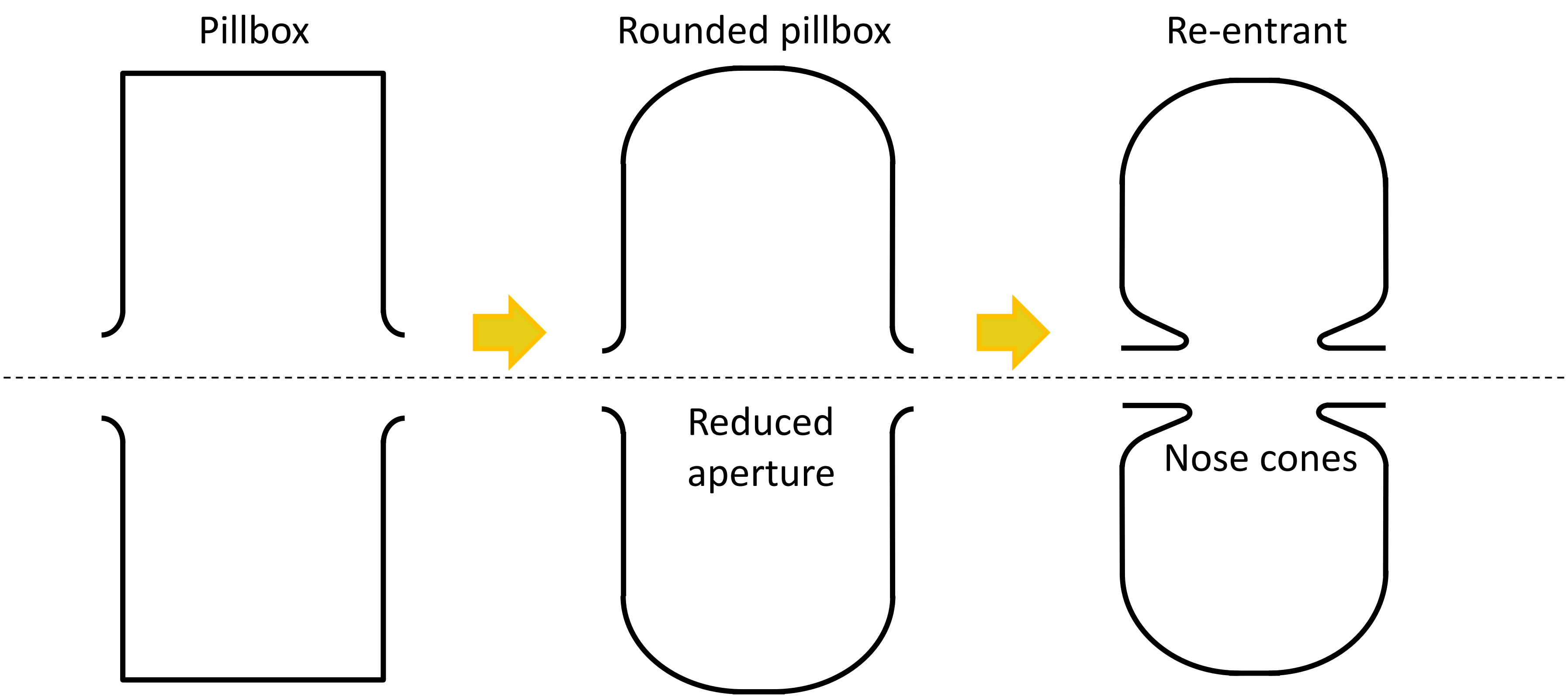
Shunt impedance (not to be confused with surface resistance) is a measure of how efficiently a cavity uses RF power to accelerate particles.

Cavity *Q* is a measure of how long the stored energy in the cavity dissipates. Q_0 is the ratio of stored energy to loss per RF cycle.

R/Q depends only on the cavity geometry.

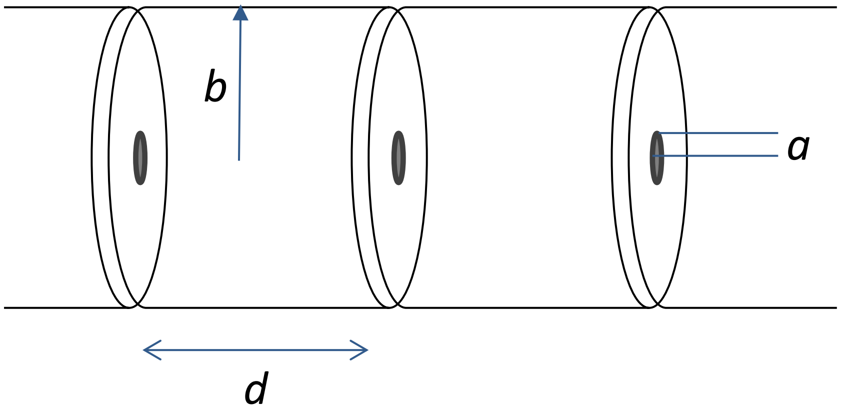
Evolution of NCRF Cavity Shapes

NCRF cavities evolve toward re-entrant shape for higher shunt impedance.



The nose cones and reduced apertures concentrate electric field lines near the axis, which increases shunt impedance, TTF and wakefield effects.

Periodically Loaded Structure



Disk-loaded structure

Typical values for TW S-band structures

f = 2.856 GHz

a = 1.1625 cm

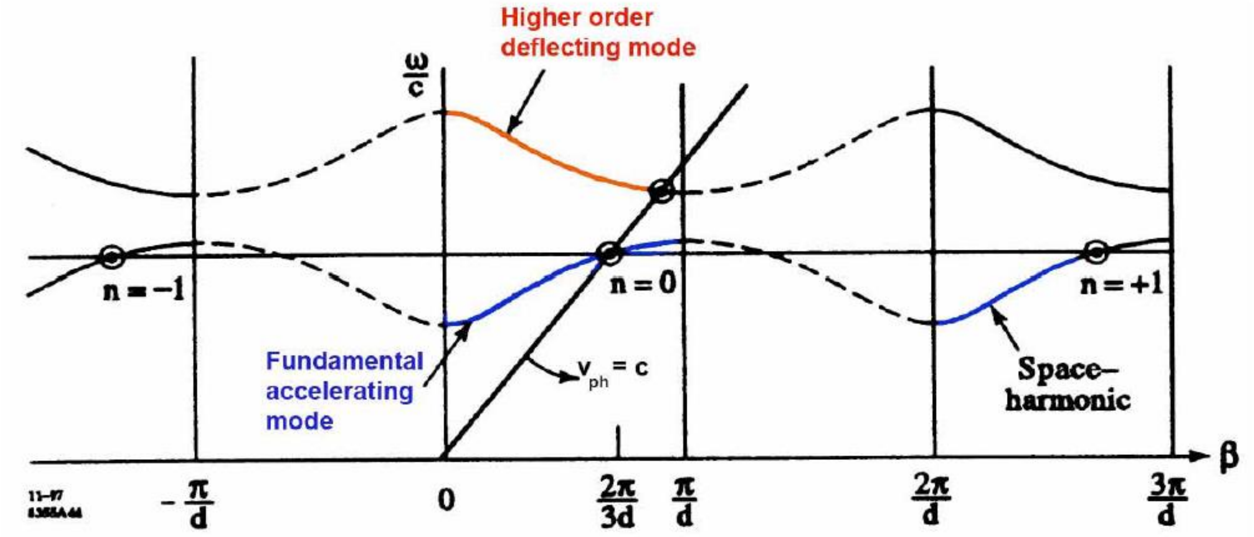
v_g = 0.013*c*

b = 4.1325 cm

d = 3.5 cm

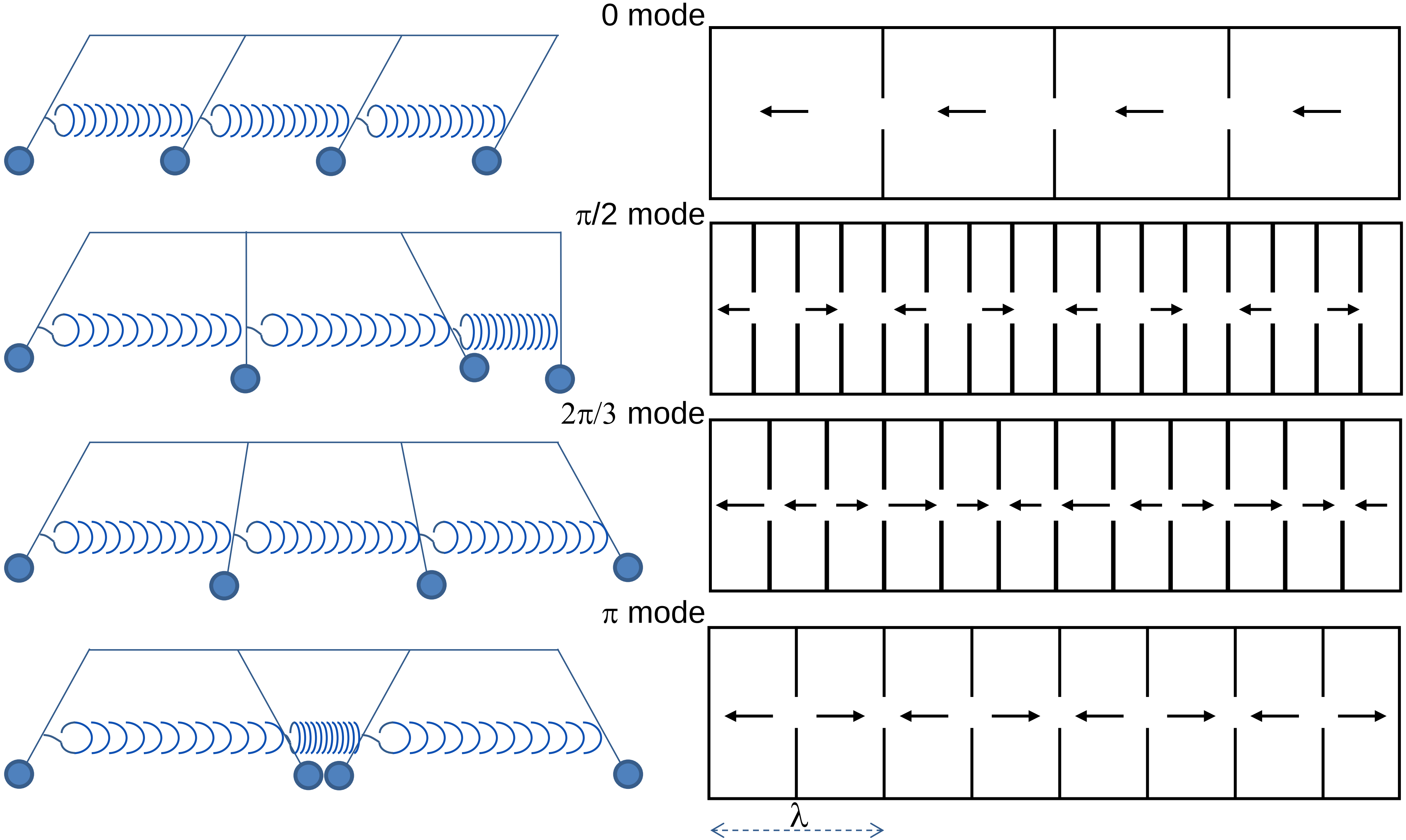
$\phi = 2\pi/3$ (120° phase advance per cell)

Brillouin diagram



Phase advance per cell

Coupled Cavity Linac



Normal-conducting RF Linac

NCRF linac technology choices

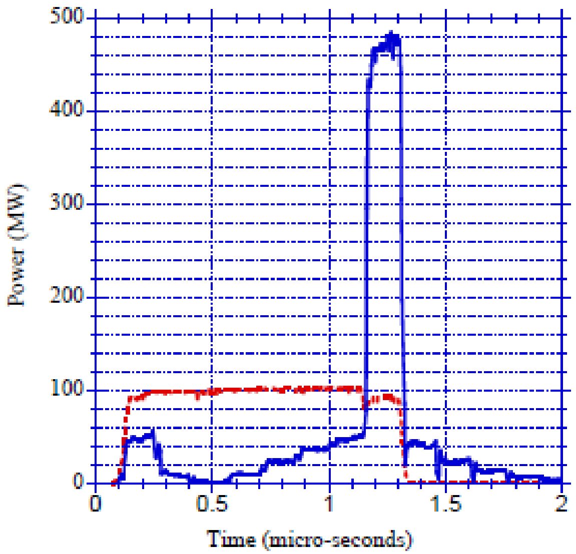
- Traveling-wave linac
  - Constant-impedance
  - Constant-gradient
- Standing-wave linac
  - Electrically coupled
  - Magnetically coupled
- Frequency

Typical operating conditions

- Room-temperature copper
- Pulsed, low repetition rate
- High gradients at high frequency
- Requiring high RF power (RF compression)

	S-Band	C-Band	X-Band
	SLAC	XFEL/Spring-8	NLC Test
Klystron	• SLAC 5045 • 65 MW, 3.5 μs, 120 Hz	• Toshiba E3746 • 50 MW, 2.5 μs, 60 Hz	• SLAC XL4 and XL5 • 50 MW, 1.5 μs, 60 Hz
Linac	• CG TW • 20 MV/m, 900 ns	• CG Choke mode • 35 MV/m, 300 ns	• 80 MV/m, 400 ns • 90MV/m, 1.2 μs single cell tests

RF power without (red) and with (blue) RF compression



Constant Impedance TW Structures

Characteristics of CI-TW structures

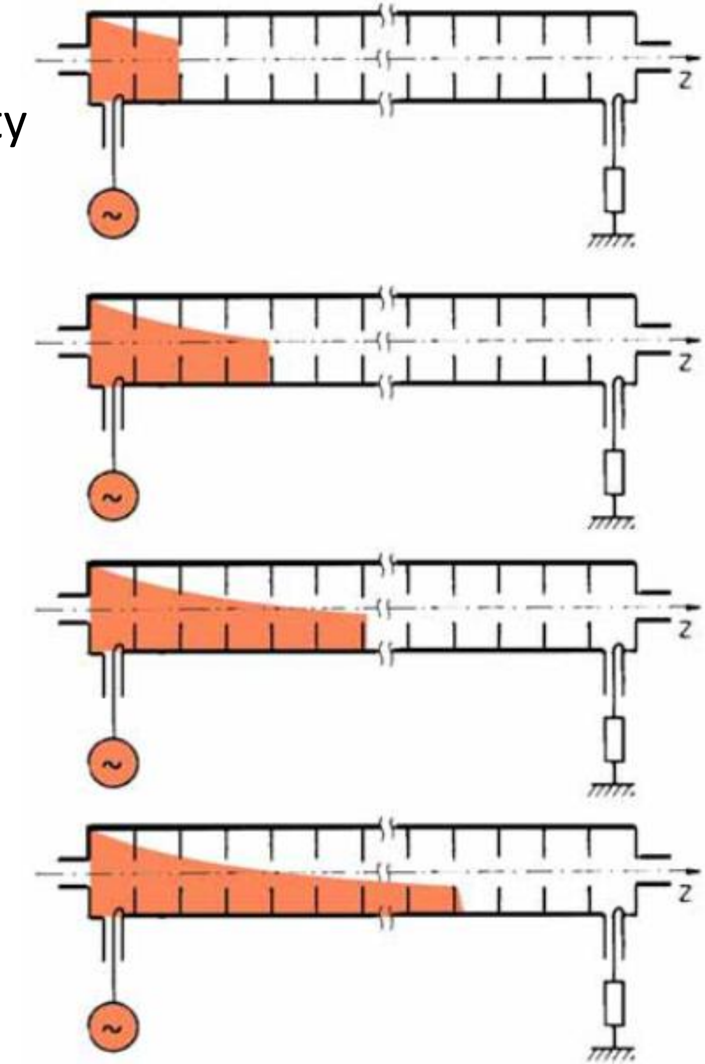
- Aperture size is the same throughout the cavity
- RF power and accelerating gradient decay exponentially along the length of the cavity

$$\frac{P_{out}}{P_{in}} = e^{-2\tau_0} \qquad \tau_0 = \alpha l$$

$$E_a(z) = E_0 e^{-\alpha z} \qquad \alpha = \frac{\omega_0}{2\upsilon_g Q}$$

- Cavity fill time

$$t_F = \frac{2Q}{\omega}\tau_0$$



Constant-Gradient TW Structure

Characteristics of CG-TW structures

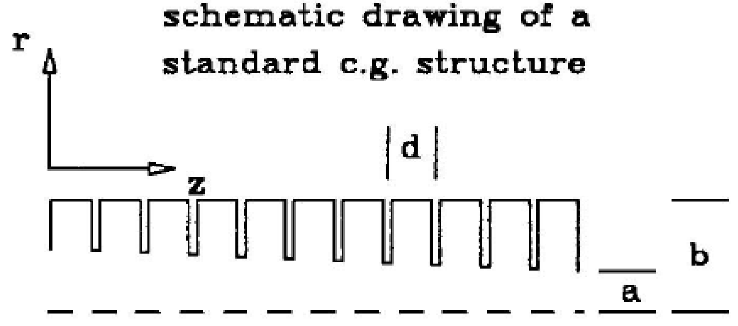
- Aperture size and attenuation decrease along the cavity length
- Attenuation depends on z

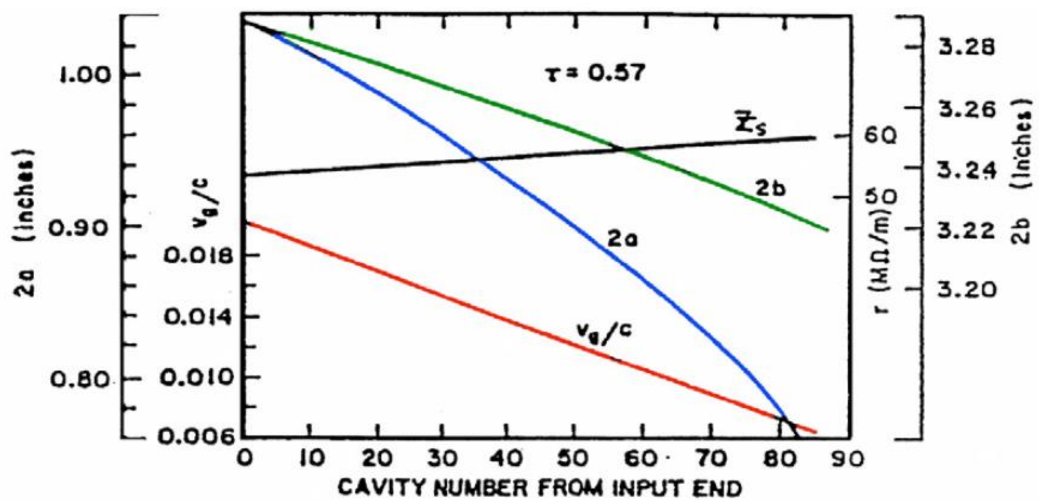
$$\frac{dP_w}{dz} = -2\alpha(z)P_w$$

$$P_w(z) = P_0\left[1 - \frac{z}{L}\left(1 - e^{-2\tau_0}\right)\right]$$

- Cavity fill time

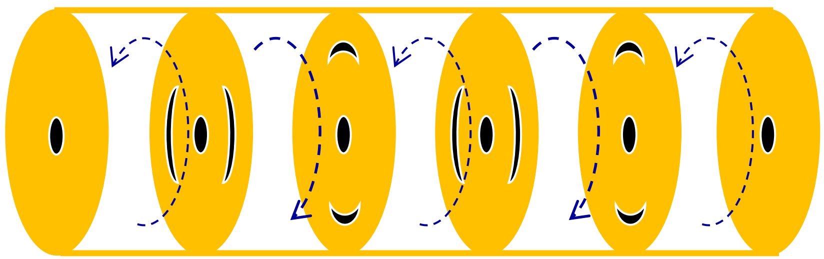
$$t_F = \frac{2Q}{\omega}\tau_0$$



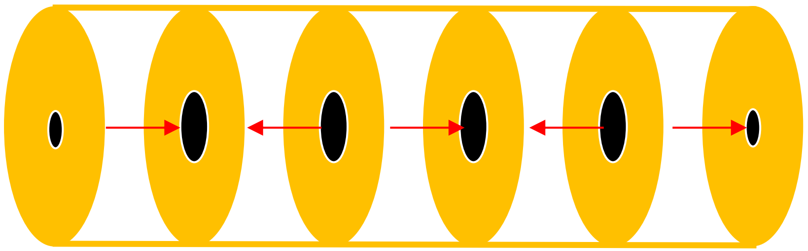


Multi-cell Standing-Wave Cavity

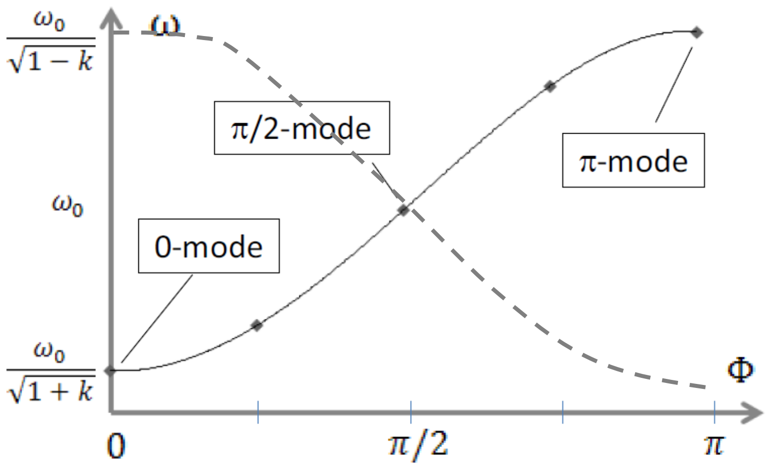
A standing wave can be decomposed into two counter-propagating waves of equal amplitude. The cell-to-cell coupling can be either magnetic (blue) or electric (red).



Magnetically coupled 5-cell cavities



Electrically coupled 5-cell cavities



k = coupling strength

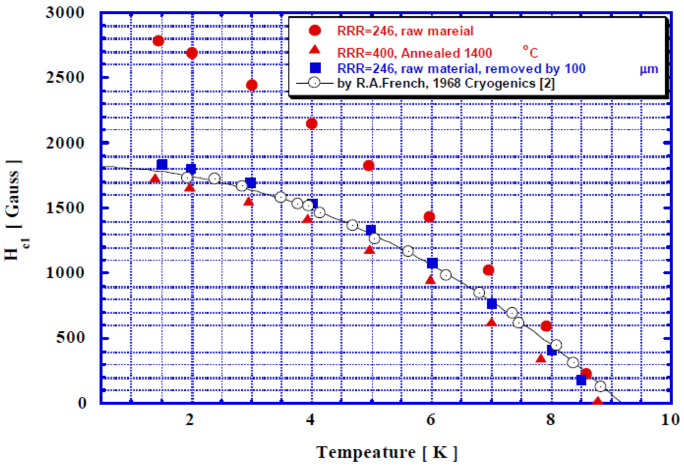
The π-mode has the highest frequency in electrically coupled cavities (solid), while the 0-mode has the highest frequency in magnetically coupled cavities (dashed).

RF Superconductivity

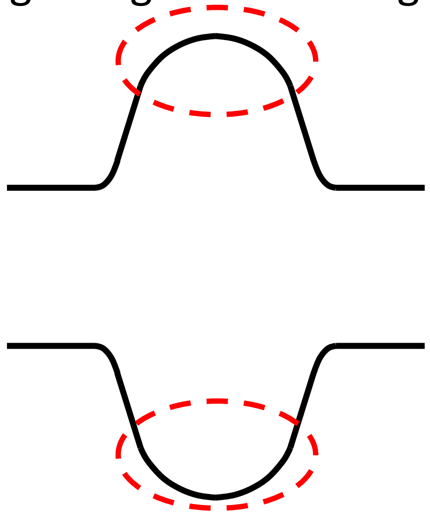
- The most common material for SCRF cavities is Nb, a type-II superconductor. Nb becomes superconducting at temperature below 9.2K (T_c).
- Superconductivity can be destroyed by magnetic fields greater than the critical field, H_c. For Nb at 2K, the critical field is 170 +/- 10 mT.
- SCRF cavities are often cooled with liquid helium at 1.9K (lambda point) or 4.2K (boiling point).
- Accelerator cavities are designed with elliptical shapes and large apertures. The main factor limiting SCRF accelerating gradients is the magnetic field. In TESLA cavities, the ratio of B_{peak} to E_{acc} is

$$\frac{B_{peak}}{E_{acc}} \approx 4.2 \frac{mT}{MV/m}$$

so the maximum gradient is ~40 MV/m.



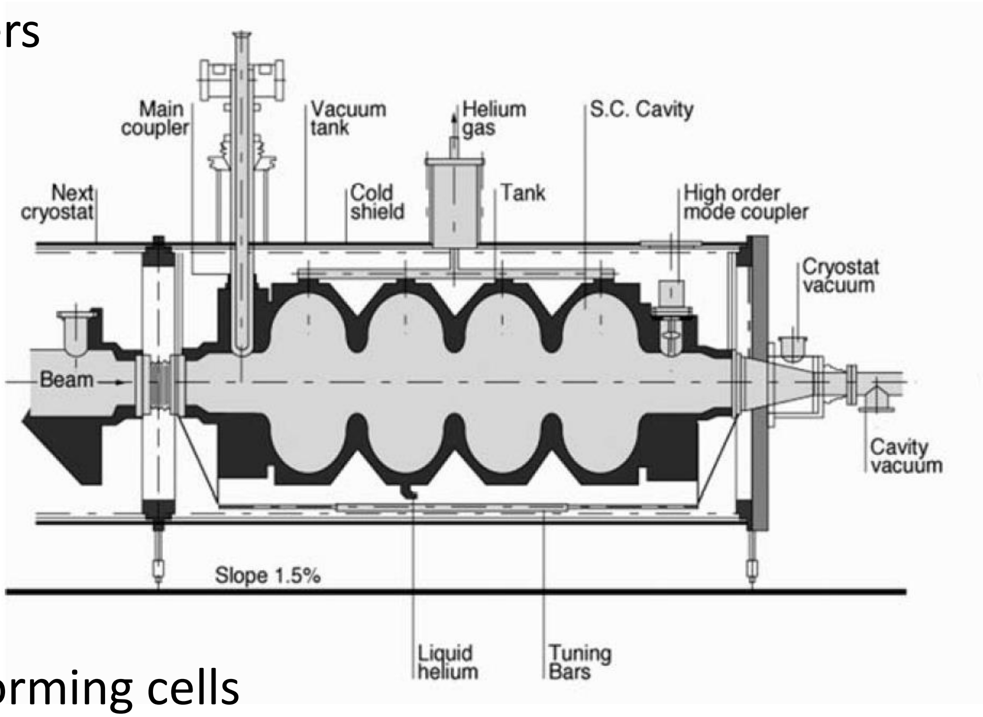
High magnetic field regions



Superconducting RF Linac

SCRF linac technologies

- Standing-wave π-mode cavities
  - Electrically coupled through the apertures
- RF power is fed into cavities via a main power coupler
- HOM is extracted via HOM couplers
- # of cells per cavity = 9 (TESLA)
- # of cavities per cryomodule = 8

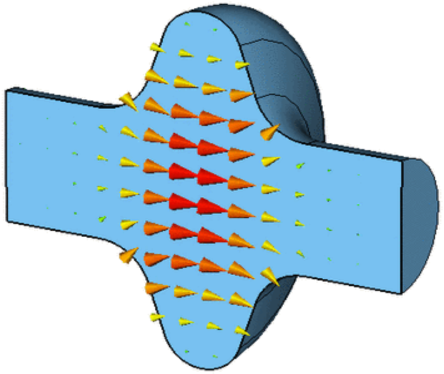


Typical operating conditions

- Liquid-helium cooled niobium
- High repetition rate or CW
- Medium-gradient
- Gradient is limited to lowest performing cells

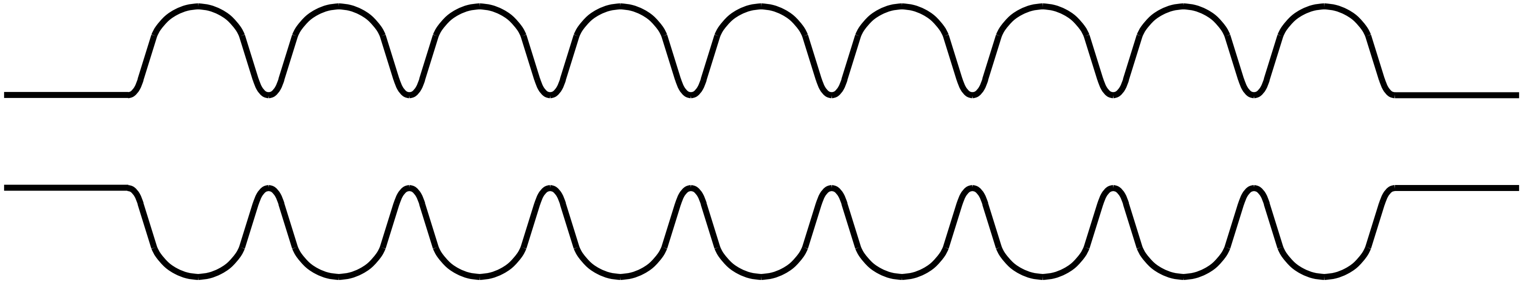
Superconducting RF Cavities

A single SCRF cell

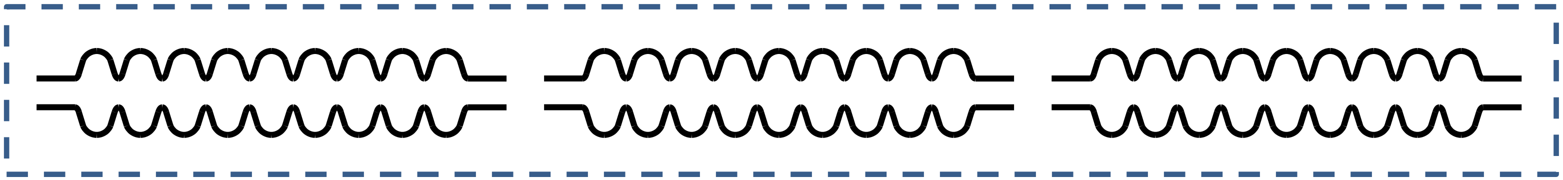


This shows the cut-away of one SCRF cell and the accelerating electric fields

9-cell cavity (π-mode)

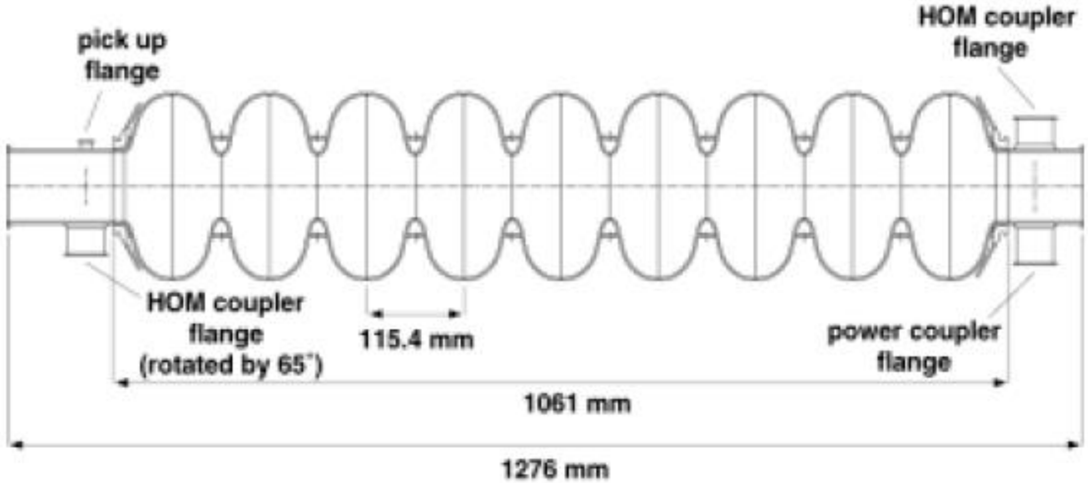


There are N cavities in a common cryo-module



TESLA 9-cell Cavity

TESLA 9-cell cavity is the most common SRF structure at 1.3 GHz.



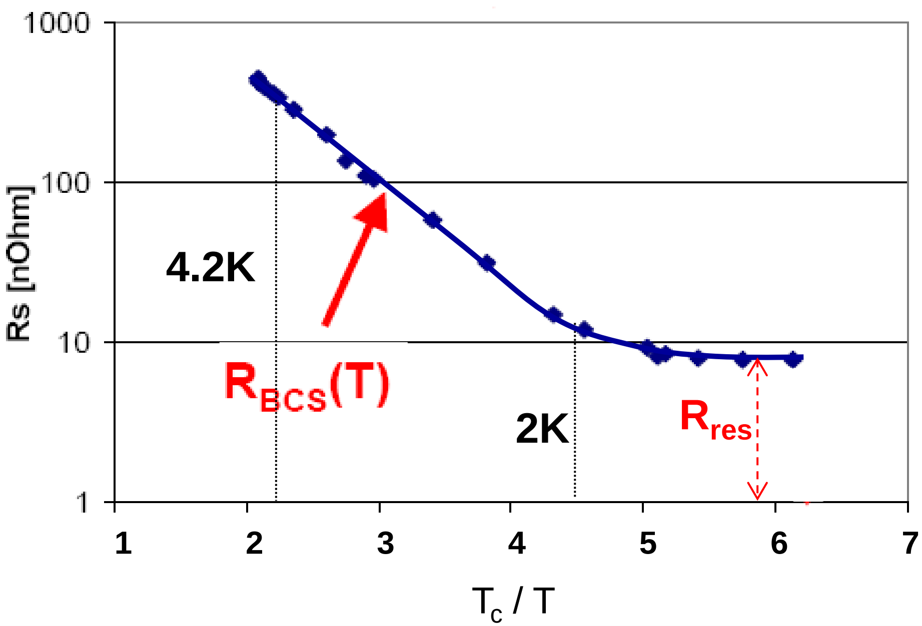
TESLA 9-cell cavity has an R/Q of 114 Ω/cell or an r/Q of ~1 kΩ/m

Surface Resistance

Surface resistance of Nb has two parts: the residual resistance and the Bardeen-Cooper-Schrieffer resistance which is temperature-dependent.

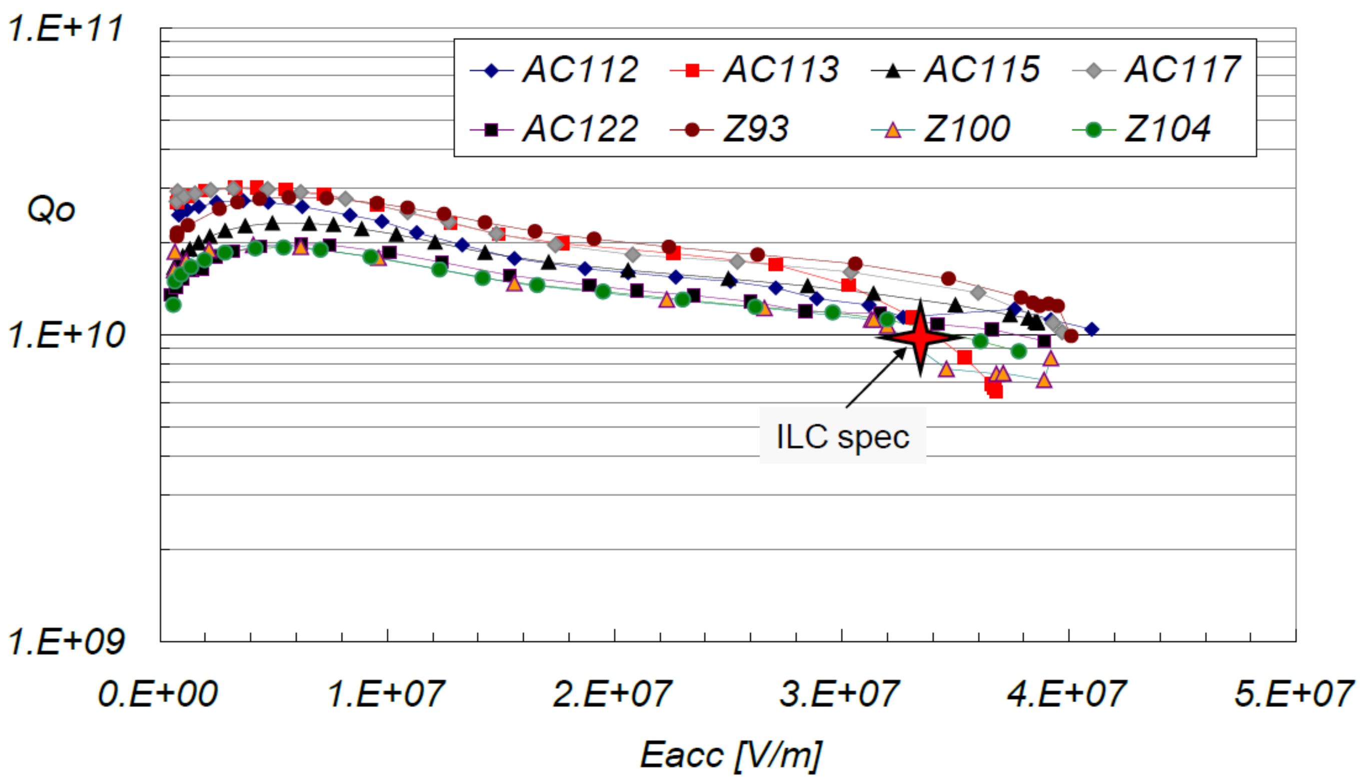
$$R_S = \underbrace{2\times 10^{-4}\Omega\left(\frac{f}{1.5GHz}\right)^2\left(\frac{1}{T}\right)\exp\left(\frac{-17.67}{T}\right)}_{R_{BCS}(T)} + R_{residual}$$

The f^2 dependence of BCS resistance requires SRF cavities with $f > 500$ MHz to be cooled at < 2K. Cavities with $f < 500$ MHz can be cooled with atmospheric helium at 4K.



Plot of surface resistance versus the ratio of T_c (9.2K) to temperature.

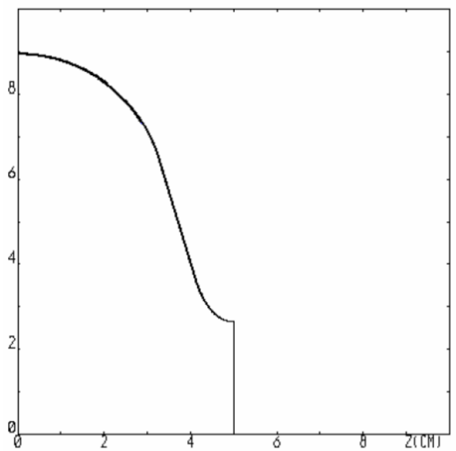
Unloaded Q_0



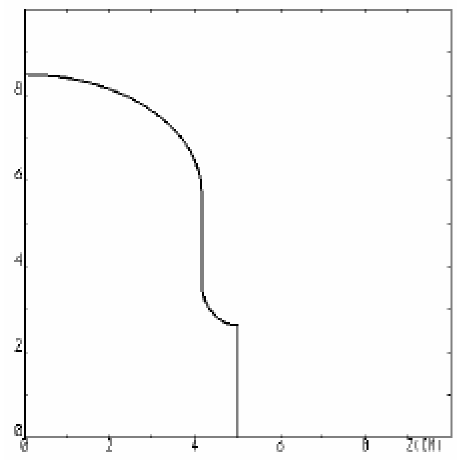
Plots of Q_0 versus E_{acc} for TESLA 9-cell cavities. The highest accelerating gradient at 40 MV/m corresponds to surface magnetic field of 168 mT.

Q_0 at $E_{acc} = 0$ is approximately 270Ω divided by the surface resistance.

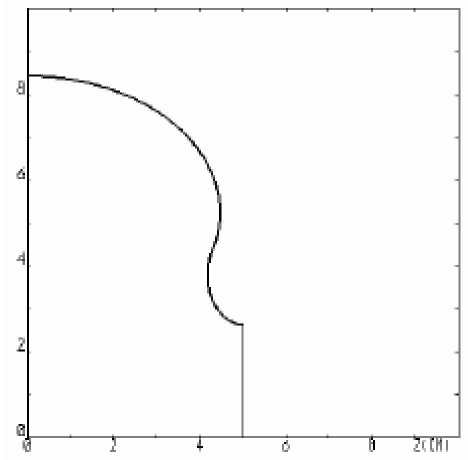
Selection of RF Cavity Shapes



Elliptical (TESLA)



Low-loss



Re-entrant

Optimized	TESLA	Low-loss	Re-entrant
E_{peak}/E_{acc}	1.98	2.36	2.30
B_{peak}/E_{acc}	4.15	3.61	3.57
R/Q (Ω)	113.8	133.7	135
G (Ω)	271	284	284.3
(R/Q)*G (Ω^2)	30,840	37,970	38,380

from
J. Sekutowicz's
Superconducting
Cavities

Strategies to Reduce Power Dissipation

Power dissipation per cell normalized to the square of voltage gain

$$\frac{P}{V_{cell}^2} = \frac{1}{\left(\frac{R}{Q}\right) Q_0} = \frac{R_S}{\left(\frac{R}{Q}\right) G}$$

Minimizing R_S

Increase the ratio of T_c to T → lower BCS resistance

$$R_{BCS} \propto f^2 \left(\frac{1}{T}\right) \exp\left(1.92 \frac{-T_c}{T}\right)$$

Bake at high temperature in nitrogen atmosphere

Electropolish to remove 70 μm of materials

Bake at low temperature under ultrahigh vacuum

Maximizing (R/Q)*G

Select low-loss or re-entrant cavity geometry

Summary

- Acceleration of charged particles along the same direction as the RF wave propagation is done in cylindrical RF cavities with TM_{010} mode.
- The RF pulse format determines which RF linac technology to use. For very short (<3 μs) RF pulse, use TW NCRF; short (<30 μs) RF pulse, use SW NCRF; long-pulse or CW, use SCRF.
- High-frequency TW linac provide the highest accelerating gradients, in ~μs RF pulses and requiring peak power >50 MW.
- L-band SW SCRF linac are often used for CW operation delivering MHz electron bunches for high-rep-rate x-ray FEL.
- Recipes exist to improve the performance of SRF cavities, in both accelerating gradients and RF efficiency.